# Meaning to Please

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**The Mathematical Intelligencer**

ISSN 0343-6993
Volume 40
Number 1

Math Intelligencer (2018) 40:68-72
DOI 10.1007/s00283-018-9780-z


Springer

# Meaning to Please

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*This is a column about the mathematical structures that give us pleasure. Usefulness is irrelevant. Significance, depth, even truth are optional. If something appears in this column, it's because it's intriguing, or lovely, or just fun. Moreover, it is so intended.*

Sometimes mathematical structures are designed to please. They are brought into this world not to serve but to charm. They live not only because they are true but also because they intrigue. They demand attention because they excite wonder and delight.

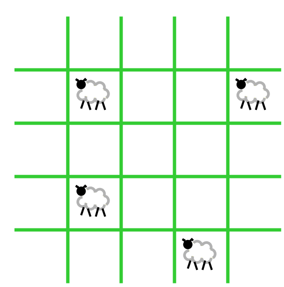
Mathematics that is pleasurable is not new. But I think that something has changed in the last hundred years or so. Mathematics created specifically to please gets more attention today. And there seem to be more mathematicians (and others) whose private and public joy has been the pleasure of their mathematical creations. It is this phenomenon—the compelling mathematical structures, the people who found them, and the society that appreciates them—that is the focus of this column.

I'll set the stage today by talking a little about mathematical structures, about mathematical pleasure, and about intention. Then I'll show you something unexpected.

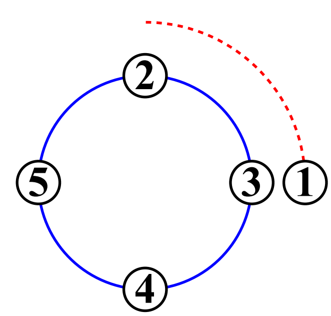
To the world, mathematical expertise is special. Still more special is the capacity to appreciate mathematical beauty. And most special of all is the intelligence and sensitivity to create beautiful mathematical structures. But special, it turns out, is not rare. I taught a course last year that was specifically for students with no required mathematical background. I asked the students to create structures. I gave them some background and schooled them in some mathematical aesthetics. They came up with some gems.

When the preliminaries are over, I'll show you three examples:

a puzzle,



a dance,

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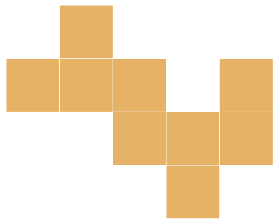
https://doi.org/10.1007/s00283-018-9780-z

and ...something else.



## Mathematical Structures

We won't deal with "mathematics." "Mathematics" is notoriously difficult to define. Philosophers and mathematicians quarrel over issues of truth, knowledge, morality, and existence.[1] Instead of "mathematics" we will discuss "mathematical structures." It turns out this is pretty easy to define. I simply tell my students:

> **A mathematical structure is anything that can be described completely and unambiguously.**

Everything we normally think of as a mathematical structure satisfies this definition. Nothing in the physical world does.

The definition is perfect for anyone with a limited mathematical background. Many remember their math classes with regret and anxiety. They are more comfortable with language. For them, this definition is simple and meaningful.

The definition says nothing about a structure's interest, beauty, or importance, nor its ability to give pleasure. That's next.

## Mathematical Pleasure

The structures in this column should be judged based on the mathematical pleasure they provide. The concept of *mathematical pleasure* is more difficult to describe. It certainly includes what we normally think of as mathematical beauty. But there is more to it. There is, for example, the pleasure of solving a problem, the pleasure of seeing a solution, and the pleasure of an intriguing question.

Robert Thomas has written that mathematical aesthetics should include the interest generated by a work.[2] That suggests an operational definition:

> **If a mathematical structure attracts interest, if the structure is played with, investigated, and explored, then it is mathematically pleasurable.**

## Intention

There is a wealth of pleasurable mathematics. We are the inheritors of three thousand years of glorious stuff. What interests me, though, is the mathematics that is *intentionally* attractive.

It's difficult to know the intention of a creator. For mathematicians before 1900, it might be impossible to be sure what they most cared about. For contemporary mathematicians, though, there is evidence. Martin Gardner's column in *Scientific American* celebrated mathematics with appeal. It introduced us to structures by John Horton Conway, Donald Knuth, and many others, which were clearly designed to tickle, confound, provoke, and amaze. The very existence of Gardner's column motivated mathematicians and nonmathematicians to dream up structures that would dazzle and intrigue.

## Where This Column Might Go

I hope to have columns about:

- the work of individual creators: Conway and Knuth, for example, and Piet Hein, and the late Raymond Smullyan
- particular genres and subgenres: take-away games, numeration systems, card tricks, etc.
- the creativity of *Nikoli*
- Luca Pacioli and *De Veribus Quantitatis*
- the games of Sid Sackson
- the box score puzzles of Jerry Butters
- dances and change-ringing
- disputes about structures, aesthetics, and history
- the U.S. Puzzle Championship
- and (whenever possible) the Latest New Thing.

And now from my students:

## A Puzzle-form

Puzzles can be mathematical structures. And they can certainly be pleasurable. But once a puzzle is solved, it loses much of its attraction. Most of us discard the sudokus we have solved (or messed up). But think of sudoku as a *puzzle-form*. Think of it as a set of rules for what constitutes a sudoku puzzle. As a puzzle-form, sudoku is a fabulously successful mathematical structure. The public has embraced it. Thousands, probably hundreds of thousands of sudoku puzzles have been created and enjoyed.

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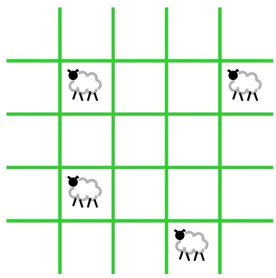
[1] My qualifications for defining mathematics are dangerously thin. I'm skeptical of truth, unreliable about knowledge, and of dubious (mathematical) morality. My strongest suit is existence.

[2] "Beauty Is Not All There Is to Aesthetics of Mathematics," *Philosophia Mathematica*, 13 September 2016

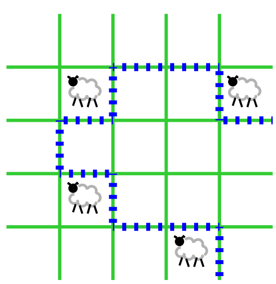
Sudoku was the invention of an American architect, Howard Garns, who published the first sudoku puzzles in 1979.[3] I am confident Garns intended his invention to give pleasure.

We are living today in a golden age of puzzles. Sudoku has been followed by a host of attractive puzzle-forms. The Japanese publisher Nikoli Co., Inc. is responsible for many of them—Shikaku, Masyu, Nurikabe, Slitherlink, and many, many others.

A year ago, my student EvaMarie Olson invented a puzzle-form she called "Save the Sheep." A save-the-sheep puzzle is a grid with sheep.

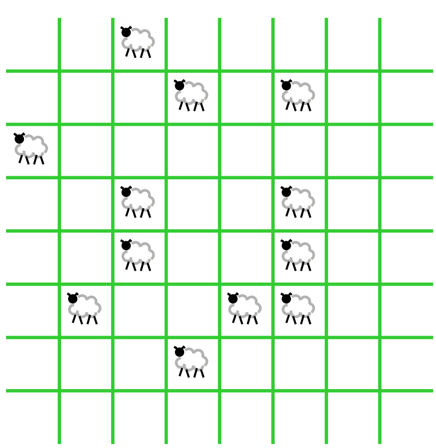


To solve the puzzle you must erect a fence with all the sheep on one side. The fence must begin and end at a side of the grid. The fence can't visit any lattice point twice. Finally, it must include exactly two sides of each sheep's square.



Fortunately, I know EvaMarie's intention. Her assignment was to create a pleasing puzzle-form.

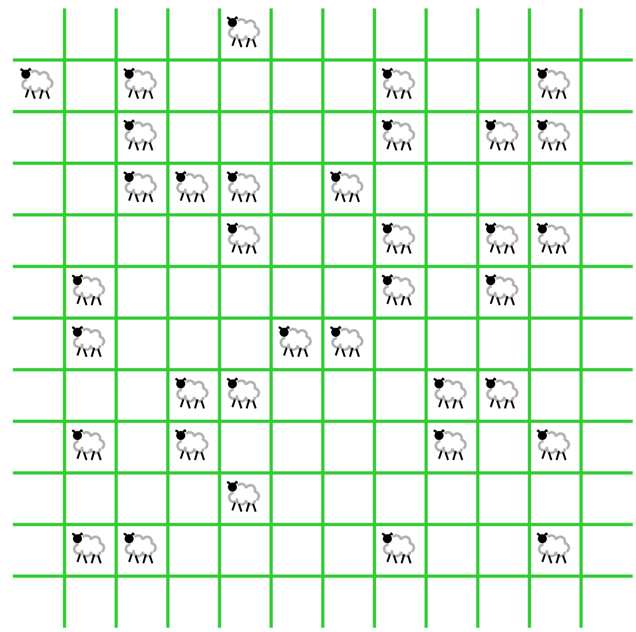
Here's a more complex save-the-sheep puzzle:



Is this puzzle-form good? How do we evaluate it?

I can't say how successful Save the Sheep is at this point. I can only say that it attracted me and it attracted me in the most important way. I wanted to make save-the-sheep puzzles. I was drawn to it in the mathematical, intellectual sense. I wanted to explore the possibilities.

Here is what I came up with, a really complex save-the-sheep puzzle that has (I hope) just one solution:



I'll post answers to these puzzles in good time, at

www.math.smith.edu/~jhenle/pleasingmath/

If you too are taken with this puzzle-form, send your save-the-sheep puzzles to me at

pleasingmath@gmail.com

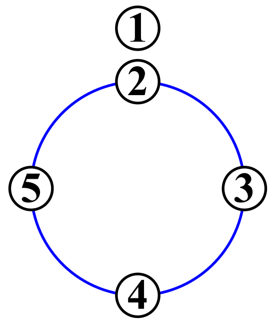
Send comments too.

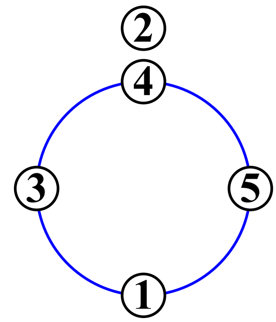
## A Dance

Reality is not a mathematical structure. But aspects of reality are mathematical. Consider dance. It's impossible to describe the moves and gestures of a dancer completely and unambiguously. On the other hand, if we restrict our attention to the physical location of the dancers at set intervals (on a grid, say) then we do have a mathematical structure. There are examples of notation systems for many dance genres, and these capture their mathematical aspects.

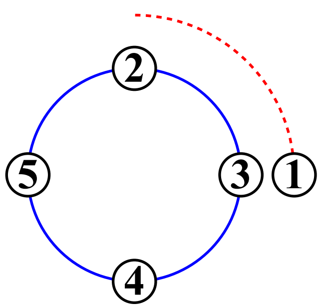
My students—Connie Adamson, Victoria Nompleggi, Haley Peterson, and Desiree Viola—invented a dance with an intriguing mathematical structure. The five dancers, conveniently named 1, 2, 3, 4, and 5, might begin like this:

[3]He called the puzzles "number place" puzzles. It was the Japanese magazine *Nikoli* that called them sudoku and that's where they subsequently took off.

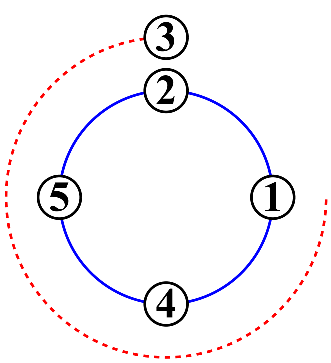



In the dance, which they called "Duck, Duck, Goose," one dancer starts outside the circle in the top position. When the dance starts, she moves clockwise one step (because she is "1").




Dancers 1 and 3 then change places, and 3 moves three places around the circle (as she is "3").




Dancers 3 and 2 now change places and the dance continues in the same way until the original formation returns.

Duck, Duck, Goose especially intrigued one of its inventors. Haley found that it took 20 steps for the original configuration to return. She wanted to know why. Starting with other configurations of the five dancers leads to different numbers of steps to return, 2, 4, 36, .... For this starting configuration it takes exactly 22 steps to return. Why 22? And why, she wanted to know, were all the numbers she was getting even?

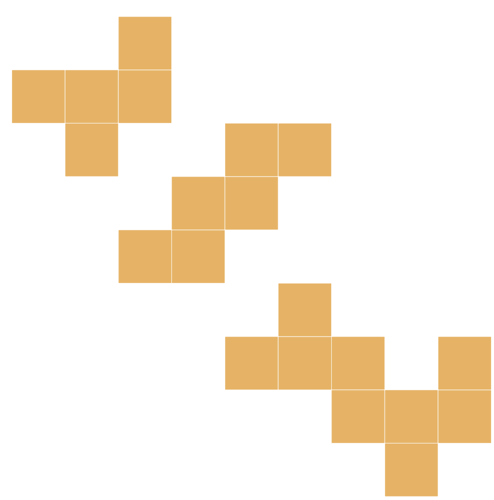
Once again we have to rely on the judgment of the few critics to view this structure (my class and myself). I found Duck, Duck, Goose sufficiently compelling to write programs to explore it. If any readers have insights, let me (and Haley) know!

## (Mathematical) Creativity Is Not so Rare

It isn't difficult to create a mathematical structure. It doesn't take extraordinary mathematical talent. But what about a great mathematical structure? What about a structure that gives mathematical pleasure?

Pleasing structures also aren't difficult to create. It's like taking great photographs. A serious photographer will take a zillion pictures. She'll throw out most of them. The best will be pretty good. One might be spectacular.

Another student in the class, Sasha Rosenthal, created what she called the "Infinite Polygon." In essence, she defined a new class of figure. Her figures are cut from a grid of unit squares along the grid lines. Sasha required that the number of sides of the figure be exactly twice its area.



I'm going to call the figures "noggles" because, as you can see, they are noggley.

There's one noggle each of sizes two and three. There's no noggle of size one, but Sasha considered a single square as an honorary noggle.

What do you do with noggles? You can put them together to form shapes, like the shapes Solomon Golomb formed with pentominoes. But the possibilities with noggles are far greater, as there are infinitely many distinct noggles. Sasha formed 2×2, 3×3, and 4×4 squares out of noggles. In each case, the noggles were all different. I found that you can form any square out of distinct noggles, but here is a challenge:

Sasha and I noticed that $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36 = 6^2$ and wondered if you could form a 6×6 square using one noggle of each of those eight sizes.

You can. An answer will be posted on the website (in good time).

The next triangular number that is also a perfect square is

$$1 + 2 + 3 + \ldots + 48 + 49 = 36^2.$$

I haven't tried that yet!

## The Payoff

My last column for *Cucina Matematica* presented a pedagogy for mathematics that prioritized enabling students to find pleasure in mathematics. The course in which Eva-Marie, Connie, Victoria, Haley, Desiree, and Sasha saved sheep, ducked geese, and nuggled noggles was an experiment in that pedagogy. There were other successes, and future columns will describe them.