Tolstoy's thesis . . . is that there is a natural law whereby the lives of human beings no less than those of nature are determined; but that men, unable to face this inexorable process, seek to represent it as a succession of free choices, to fix responsibility for what occurs upon persons endowed by them with heroic virtues or heroic vices, and called by them 'great men'. What are great men? They are ordinary human beings, who are ignorant and vain enough to accept responsibility for the life of society, individuals who would rather take the blame for all the cruelties, injustices, disasters justified in their name, than recognize their own insignificance and impotence in the cosmic flow which pursues its course irrespective of their wills and ideals.

–Sir Isaiah Berlin

Conversation

Tolstoy and History

The calculus is more than a mathematical theory, more than the language of science; it is an integral part of human culture. Like scientific ideas today, such as relativity and chaos, calculus is part of the vocabulary of all educated people.

***War and Peace*, the great historical novel of Count Lev Nikolaevitch Tolstoy, is simultaneously literature and philosophy. Tolstoy's long-held views of history, in some sense, drive the book. For us, the most interesting exposition of his views comes at the beginning of Book XI. Here, uses he the language of calculus to make his point.**

Laboratory: Infinitesimals and History

Your challenge in this laboratory is comment on Tolstoy's use of calculus. We include here a long uninterrupted passage for you to examine. Your "comment" can be anything from a few sentences to an essay.

A word of explanation: Tolstoy refers often to "infinitesimals." The calculus of Newton and Leibniz was originally expressed with infinitesimals, limits coming later in the nineteenth century. For example, while modern mathematicians would say that as Δx

approaches 0, the difference quotient,

$$\frac{f(a+\Delta x)-f(x)}{\Delta x},$$

approaches $f'(a)$, mathematicians of Tolstoy's time would say that for infinitesimal Δx, the quotient is infinitesimally close $f'(a)$. Some even said that it *is* $f'(a)$.

The difference is more acute when discussing integrals. We say today that the integral, $\int_a^b f(x)dx$, is the *limit* of sums,

$$f(a)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(b)\Delta x,$$

whereas earlier the integral was said to be the infinite sum of infinitely many infinitesimals $f(x)\Delta x$. The "dx" of the integral notation, called the *differential*, was considered to be an infinitesimally small quantity.

The excerpt that follows is the entire first chapter of Book XI of *War and Peace*. It is from the celebrated translation of Louise and Aylmer Maude.

> **Absolute continuity of motion is not comprehensible to the human mind. Laws of motion of any kind only become comprehensible to man when he examines arbitrarily selected elements of that motion; but at the same time, a large proportion of human error comes from the arbitrary division of continuous motion into discontinuous elements. There is a well-known, so-called sophism of the ancients consisting in this, that Achilles could never catch up with a torroise he was following, in spite of the fact that he travelled ten times as fast as the tortoise. By the time Achilles has covered the distance that separated him from the tortoise, the tortoise has covered one-tenth of that distance ahead of him: when Achilles has covered that tenth, the tortoise has covered another one-hundredth, and so on for ever. This problem seemed to the ancients insoluble, The absurd answer (that Achilles could never overtake the tortoise) resulted from this: that motion was arbitrarily divided into discontinuous elements, whereas the motion both of Achilles and of the tortoise was continuous.**
>
> **By adopting smaller and smaller elements of motion we only approach a solution of the problem, but never reach it. Only when we have admitted the conception of the infinitely small, and the resulting geometrical progression with a common ratio of one-tenth, and have found the sum of this progression to infinity, do we reach a solution of the problem.**
>
> **A modern branch of mathematics, having achieved the art of dealing with the infinitely small, can now yield solutions in other more complex problems of motion, which used to appear insoluble.**
>
> **This modern branch of mathematics, unknown to the ancients, when dealing with problems of motion, admits the conception of the infinitely small, and so conforms**

to the chief condition of motion (absolute continuity) and thereby corrects the inevitable error which the human mind cannot avoid when dealing with separate elements of motion instead of examining continuous motion.

In seeking the laws of historical movement just the same thing happens. The movement of humanity, arising as it does from innumerable arbitrary human wills, is continuous.

To understand the laws of this continuous movement is the aim of history. But to arrive at these laws, resulting from the sum of all those human wills, man's mind postulates arbitrary and disconnected units. The first method of history is to take an arbitrarily selected series of continuous events and examine it apart from others, though there is and can be no *beginning* to any event, for one event always flows uninterruptedly from another.

The second method is to consider the actions of some one man—a king or a commander—as equivalent to the sum of many individual wills; whereas the sum of individual wills is never expressed by the activity of a single historic personage.

Historical science in its endeavour to draw nearer to truth continually takes smaller and smaller units for examination. But however small the units it takes, we feel that to take any unit disconnected from others, or to assume a *beginning* of any phenomenon, or to say that the will of many men is expressed by the actions of any one historical personage, is in itself false.

It needs no critical exertion to reduce utterly to dust any deductions drawn from history. It is merely necessary to select some larger or smaller unit as the subject of observation—as criticism has every right to do, seeing that whatever unit history observes must always be arbitrarily selected.

Only by taking an infinitesimally small unit for observations (the differential of history, that is, the individual tendencies of men) and attaining to the art of integrating them (that is, finding the sum of these infinitestimals) can we hope to arrive at the laws of history.

The first fifteen years of the nineteenth century in Europe present an extraordinary movement of millions of people. Men leave their customary pursuits, hasten from one side of Europe to the other, plunder and slaughter one another, triumph and are plunged in despair, and for some years the whole course of life is altered and presents an intensive movement which first increases and then slackens. What was the cause of this movement, by what laws was it governed? asks the mind of man.

The historians, replying to this question, lay before us the sayings and doings of a few dozen men in a building in the city of Paris, calling these sayings and doings 'the Revolution'; then they give a detailed biography of Napoleon, and of certain people favourable or hostile to him; tell of the influence some of these people had on others, and say: That is why this movement took place and those are its laws.

But the mind of man not only refuses to believe this explanation, but plainly says that this method of explanation is fallacious, because in it a weaker phenomenon is taken as the cause of a stronger. The sum of human wills produced the Revolution and Napoleon, and only the sum of those wills first tolerated and then destroyed them.

'But every time there have been conquests there have been conquerors; every time there has been a revoltuion in any state there have been great men,' say history. And indeed, every time conquerors appear there have been wars, human reason replies, but this does not prove that the conquerors caused the wars and that it is possible to find the laws of a war in the personal activity of a single man. Whenever I look at my watch and its hands point to ten, I hear the bells of the neighbouring church; but I have no right to assume that because the bells begin to ring when the hands of the watch reach ten, the movement of the bells is caused by the position of the hands of the watch.

Whenever I see the movement of a locomotive I hear the whistle and see the valves opening and wheels turning; but I have no right to conclude that the whistling and the turning of wheels are the cause of the movement of the engine.

The peasants say that a cold wind blows in late spring because the oaks are budding, and really every spring cold winds do blow when the oak is budding. But though I do not know what causes the cold winds to blow when the oak-buds unfold, I cannot agree with the peasants that the unfolding of the oak-buds is the cause of the cold wind, for the force of the wind is beyond the influence of the buds. I see only a coincidence of occurrences such as happens with all the phenomena of life and I see that however much and however carefully I observe the hands of the watch, and the valves and wheels of the engine, and the oak, I shall not discover the cause of the bells ringing, the engine moving, or of the winds of spring. To do that I must entirely change my point of view and study the laws of the movement of steam, of the bells, and of the wind. History must do the same. And attempts in this direction have already been made.

To study the laws of history we must completely change the subject of our observation, must leave aside kings, ministers, and generals, and study the common, infinitesimally small elements by which the masses are moved. No one can say in how far it is possible for man to advance history; but it is evident that only along that path does the possibility of discovering the laws of history lie; and that as yet not a millionth part as much mental effort has been applied in this direction by historians as has been devoted to describing the actions of various kings, commanders, and ministers and propounding reflections of their own concerning these actions.

p. 1 Comments on Lab: Infinitesimals and History

The interested student should read Sir Isaiah Berlin's essay, ***The Hedgehog and the Fox*** on Tolstoy's view of history (Touchtone Books, Simon and Schuster, [1978?], or an abridged version recently published by Phoenix, ***Tolstoy and History***, 1996).

Another surprising use of the language of calculus can be found in the correspondence

of Karl Marx and Friedrich Engels. See Dirk Struik's "Marx and Mathematics", in *A Centenary of Marxism*, Science and Society, 1948.

Tolstoy could have used limits as his metaphor instead of infinitesimals. Just as no political figure controls our destiny, no term in a sequence determines the limit. What is the effect, for example, of changing the value of a term, say, a_{29} on the limit of the sequence $\{a_n\}$?

Ironically, a very modern piece of mathematics, the theory of chaos, has often been used as a model of the *positive* power of individual acts. In the most often cited example, chaos theory suggests how the motion of a butterfly's wing might trigger a tornado a continent away. Which is correct, then? How important are single acts in this universe? Can mathematics answer the question?